Generalized Correlation for Friction Loss in Drag Reducing Polymer Solutions

The definition of the pipe flow friction factor has been extended to include the effect of fluid viscoelastic properties on energy dissipation in turbulent tube flow. The resulting friction factor includes a characteristic fluid relaxation time, which can be determined directly from rheological measurements, and reduces to the usual Fanning friction factor for inelastic fluids. The use of this more general friction factor enables turbulent tube flow data for both fresh and shear degraded "concentrated" drag reducing polymer solutions of various concentrations in various tube sizes to be correlated by the usual f vs. N_{Re} relation for Newtonian fluids in smooth tubes.

RON DARBY and
HSUN-FU D. CHANG

Department of Chemical Engineering
Texas A&M University
College Station, TX 77843

SCOPE

It is well known that the addition of small amounts (measured in parts per million) of high molecular weight polymeric additives to a liquid can reduce the friction loss in turbulent pipe flow by as much as 80% below that exhibited by the solvent alone at the same flow rate. This phenomenon has been studied extensively, and a number of extensive review papers have been written on the subject (e.g., Lumley, 1973; Patterson et al., 1968; Hoyt, 1972; Berman, 1978).

Despite the large volume of work on this subject, a generalized expression or correlation relating the turbulent friction loss characteristics of these solutions to basic, readily measureable, rheological properties has not been obtained. Although it is generally agreed that turbulent drag reduction is a consequence of the viscoelastic nature of the polymer solutions, experimental characterization of these properties is very difficult because of the very low concentrations of the solutions. Nonetheless, it has been shown that these solutions are definitely nonlinear viscoelastic fluids (e.g., Darby, 1970; Bruce and Schwarz, 1970; Tsai and Darby, 1978; Argumedo et al., 1978).

The mechanism by which drag reduction occurs has, likewise, not been firmly established. Various approaches to an explanation of drag reduction have been taken, such as reduced energy dissipation (Patterson and Zakin, 1968), modified transient shear response (Hansen, 1973; Barnes et al., 1971; Townsend, 1973), boundary layer thickening (Wang, 1972; Elata et al., 1966; Seyer and Metzner, 1969; Rollin and Seyer, 1972), and resistance to extensional flow (Metzner and Metzner, 1970; Baid and Metzner, 1977). The approaches range from purely hypothetical to essentially rearrangement of turbulent flow data. However, none of them have resulted in a method of quantitatively predicting the pressure drop for a given solution from fundamental measurable physical properties of these solutions.

The objective of this study is to relate the friction loss in turbulent drag reducing polymer solutions to measurable nonlinear viscoelastic properties of these solutions, in a generalized form which is applicable to non-Newtonian viscoelastic as well as Newtonian fluids, in a wide range of tube sizes. A model which describes the non-Newtonian properties of the solutions is utilized, and the mechanism of drag reduction is modeled by the influence of viscoelastic properties on the rate of energy dissipation in the locally oscillating dissipative turbulent eddies. The model results in a generalized definition of the friction factor, which enables data for fresh and degraded polymer solutions of various concentrations in a range of tube sizes, as well as Newtonian fluids, to be reduced to a common correlation.

CONCLUSIONS AND SIGNIFICANCE

Based on a model for the rate of energy dissipation in a locally oscillating viscoelastic fluid element, a generalized viscoelastic friction factor has been defined which includes the effect of a characteristic fluid time constant on the friction loss in turbulent flow. This friction factor reduces to the usual Fanning friction factor for Newtonian nondrag reducing fluids. The use of this generalized friction factor enables data for "concentrated" drag reducing polymer solutions of three different concentrations, both freshly prepared and shear-degraded, in a wide range of tube sizes, to be reduced to the usual friction factor vs. solvent Reynolds number correlation for Newtonian fluids in smooth tubes. The viscous and elastic (time constant) parameters of the solutions which are required for the generalized correlation can be obtained directly from a knowledge of the apparent viscosity function of the solution.

EXPERIMENTAL

Concentrations of 100, 250, and 500 wppm of Separan AP-30, a partially hydrolyzed polyacrylamide manufactured by Dow Chemical Co., in distilled water were the object of this study. This additive is a very popular drag reducer for aqueous turbulent flows and, according to Hoyt (1972), exhibits maximum drag reduction at a concentration somewhat below 100 wppm. Variations in the drag reducing effectiveness of solutions made with the same commercial polymer may occur due to slight variations in molecular weight distribution from batch to batch, aging, method of formulating the solution, purity of the water especially with respect to the presence of certain ions, etc. According to the classification of Patterson et al. (1968), these solutions would be considered to be "concentrated" and would be classed as strong drag reducers.

Tube flow data, consisting of pressure gradient vs. flow rate, were ob-

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Correspondence concerning this paper should be addressed to R. Darby.

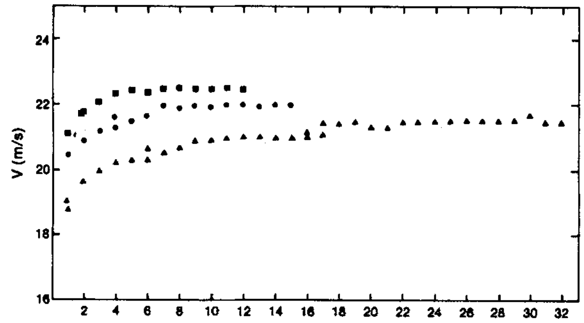

Figure 1. Typical shear history during degradation. ▲—500 wppm; ●—250 wppm; ■—100 wppm.

tained for both fresh and shear degraded solutions of the above concentrations, in stainless steel tubes with diameters of 0.178, 0.216, 0.406, 0.467, 0.616, 0.704 and 1.021 cm. Two different flow arrangements were used. For diameters above 0.5 cm, horizontal tubes with piezometric pressure taps located a minimum of 150 diameters from the ends were used. For the smaller tubes, a vertical arrangement was used with the pressure drop being obtained from the measured pressure in an upstream reservoir. In the latter case, two tubes of different lengths for each diameter were used, and the pressure gradient was obtained from the difference in pressure drop and the difference in lengths. All tube arrangements were checked and calibrated with distilled water. Further equipment details and procedures for data reduction are given by Chang (1982). The range of data covered both laminar and turbulent flow, corresponding to solvent Reynolds numbers from about 10^2 to 10^5.

The solutions exhibit pronounced nonlinear viscoelastic properties, similar to those previously reported by Tsai and Darby (1978), Darby (1970), and others. Apparent viscosity and first normal stress difference vs. shear rate were measured for all solutions, both fresh and shear degraded. Viscosity data were obtained over a shear rate range of 10^{-2} to 10^6 s^{-1} by using four different instruments. The details of these measurements and the equipment used were given by Chang and Darby (1983).

Shear degradation of the solutions was achieved by repeatedly cycling them through a 0.460 cm I.D., 0.511 m long tube at a constant reservoir pressure of 515 kPa (60 psig). The flow rate was measured for each pass, and the fluid was recycled until the flow rate no longer changed after 8 to 12 passes.

TURBULENT FLOW AND RHEOLOGICAL PROPERTIES

A typical shear history is shown in Figure 1. As seen, the velocity increases with successive cycles at constant stress, leveling off after 10–12 cycles. This indicates that the degree of drag reduction increases with shear degradation. Although this is contrary to expectations, a similar effect has been previously noted by Ng and Hartnet (1979).

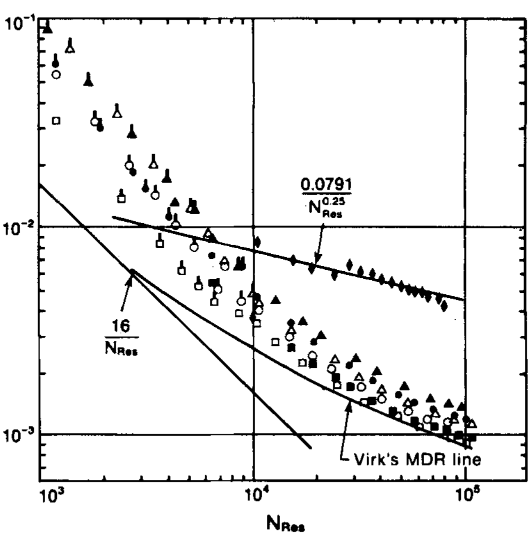

Figure 3. Fanning friction factor vs. solvent Reynolds number. Symbols same as in Figure 2. Symbols without tic are for 0.406 cm I.D. tube. Symbols with tic are for 0.467 cm I.D. tube.

The drag reducing properties of the solutions are shown in Figures 2–4 as the Fanning friction factor vs. solvent Reynolds number. The use of the Reynolds number based on the solvent viscosity provides a direct indication of the degree of drag reduction, which is defined as a reduction in pressure drop due to the addition of the polymer relative to that for the pure solvent at the same flow rate, i.e., the same solvent Reynolds number. The solvent (water) data follow the familiar Hagen-Poiseuille law for laminar flow, and the classical Blasius equation for turbulent flow. As expected for such solutions, the friction factor is greater than that of the solvent in laminar flow, but is as much as 70% below that of the solvent in turbulent flow. The 100 wppm sheared solutions show the greatest degree of drag reduction, approaching the maximum drag reduction asymptote of Virk (1975).

The nonlinear viscoelastic rheological properties of the solutions are shown in Figures 5 and 6 (from Chang and Darby, 1983). The apparent viscosity data in Figure 5 show that the effect of shear degradation is to lower the zero-shear viscosity by about one order of magnitude, although the high shear limiting viscosity is virtually unaffected. Figure 6 shows the first normal stress difference function for the solutions, which is commonly assumed to be in-

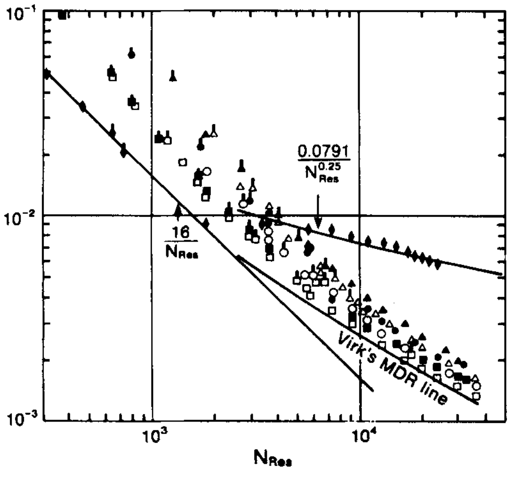

Figure 2. Fanning friction factor vs. solvent Reynolds number. ▲ 500 wppm fresh; ● 250 wppm fresh; ■ 100 wppm fresh; △ 500 wppm sheared; ○ 250 wppm sheared; □ 100 wppm sheared; ◆ water. Symbols without tic are for 0.178 cm I.D. tube. Symbols with tic are for 0.216 cm I.D. tube.

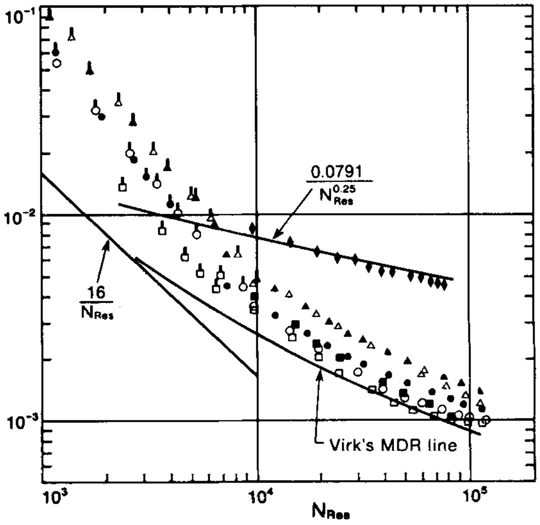

Figure 4. Fanning friction factor vs. solvent Reynolds number. Symbols same as in Figure 2. Tube diameter 0.460 cm I.D.

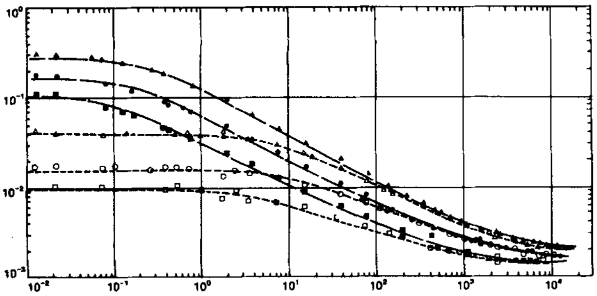

Figure 5. Apparent viscosity vs. shear rate. Symbols same as in Figure 2.

dicative of elastic character. As seen, the effect of shear degradation is to reduce the normal stress difference at all shear rates by about the same factor.

A constitutive equation has been proposed by Chang and Darby (1983) which accurately represents these data. The apparent viscosity function predicted by the equation is

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1 + \zeta^2\dot\gamma^2)^\Omega} \tag{1}$$

and the first normal stress difference function is:

$$\psi_1 = \frac{\tau_{11} - \tau_{22}}{\dot\gamma} = \frac{2(\eta_0 - \eta_\infty)\theta}{(1 + \zeta^2\dot\gamma^2)^{2\Omega+\beta}} \tag{2}$$

These functions involve six parameters, or material properties. Four of them (η_0, η_∞, ζ and Ω) serve to define the viscosity function, although all four of these properties also appear in the normal stress function, along with the remaining two (θ and β). The parameter ζ is a characteristic time constant of the fluid, and Ω is related to the flow index n in the power law region by $n = (1\text{-}2\Omega)$. The time constant ζ is the reciprocal of the shear rate at the point where the power law region of the viscosity curve intersects with η_0. The physical significance of the limiting viscosities η_0 and η_∞ is evident. Values of the six material properties are shown in Table 1 for each of the fresh and sheared solutions. The lines on Figure 5 represent Eq. 1, and the long dashed lines on Figure 6 represent Eq. 2, with the parameter values from Table 1.

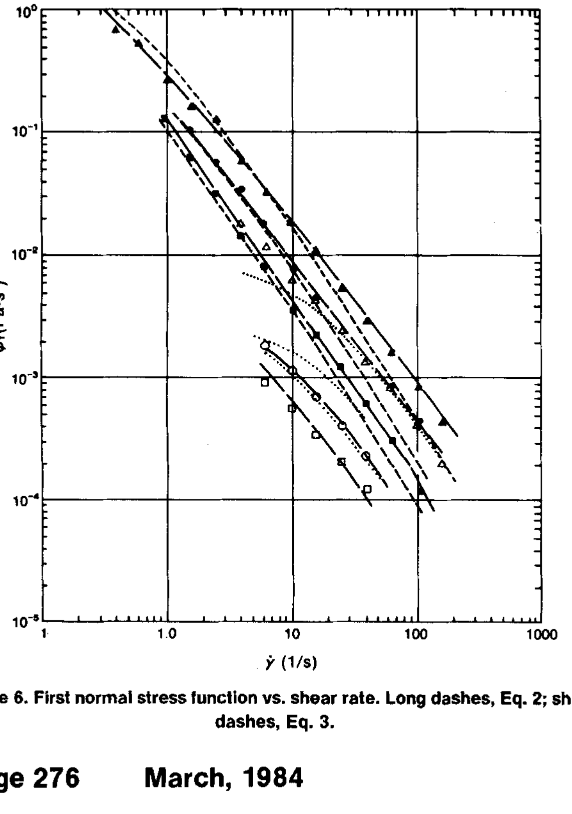

Figure 6. First normal stress function vs. shear rate. Long dashes, Eq. 2; short dashes, Eq. 3.

TABLE 1. VALUES OF RHEOLOGICAL PARAMETERS FOR SEPAREN AP-30 SOLUTIONS

Solution	η_0 (Pa·s) × 10	ζ (s)	Ω	η_∞ (Pa·s) × 1,000	θ (s)	β
100 ppm (fresh)	1.113	11.89	0.266	1.30	21.6	0.213
250 ppm (fresh)	1.714	6.67	0.270	1.40	7.10	0.133
500 ppm (fresh)	3.017	3.53	0.300	1.70	2.43	0.04
100 ppm (sheared)	0.098	0.258	0.251	1.30	0.107	0.104
250 ppm (sheared)	0.169	0.106	0.270	1.40	0.0091	0.137
500 ppm (sheared)	0.397	0.125	0.295	1.70	0.187	0.139

Although the viscous and elastic material properties are distinct, there is a strong interaction between the apparent viscosity and normal stress functions, as indicated the the number of parameters common to Eqs. 1 and 2. This relationship has been elaborated on by Abdel-Khalik et al. (1974), who proposed the following semi-empirical formula relating the normal stress and viscosity functions;

$$\psi_1 = \frac{4K}{\pi}\int_0^\infty \frac{\eta(\dot\gamma) - \eta(\dot\gamma')}{\dot\gamma'^2 - \dot\gamma^2}\, d\dot\gamma' \tag{3}$$

where $K = 2$ for solutions and 3 for melts. The values predicted by Eq. 3 are shown as the dotted lines on Figure 6. Although the agreement is not perfect, it is seen to be quite reasonable. The point to be noted is that material parameters which influence elastic behavior may also influence viscous behavior and vice versa.

ENERGY DISSIPATION MODEL

In tube flow, wall drag, pressure drop and friction loss are all synonymous with the rate of energy dissipation. Quantitatively, the relationships are:

$$\dot e = \frac{\tau_w(\pi DL)v}{\rho V} = \frac{-\Delta PQ}{\rho V} = \left(\frac{2fL\rho v^2}{D}\right)\left(\frac{\pi D^2 v}{4}\right)\left(\frac{4}{\pi D^2 L\rho}\right) = \frac{2fv^3}{D} \tag{4}$$

where $\dot e$ is the mean rate of energy dissipation per unit mass of fluid, and f is the Fanning friction factor. In time dependent deformations, energy is stored by elastic properties and dissipated by viscous properties. Since turbulence is locally time dependent, and polymeric fluids are viscoelastic, it is logical that energy which is stored by elastic properties would represent a reduction in the amount of energy which would otherwise be dissipated by viscous forces, i.e., drag reduction.

To illustrate, we consider a linear viscoelastic fluid and the simplest possible model for a time dependent deformation, e.g., a local element undergoing a small amplitude oscillating shear deformation in a large body of fluid. If the frequency of oscillation of the element is ω and the amplitude of the oscillating velocity is v_o, a shear wave will be propagated by the oscillation. The equations of motion can be solved to give the mean rate of energy dissipation per unit mass of fluid as follows (Darby, 1972):

$$\dot e = \frac{1}{\rho}\left[Re(\tau)Re(\dot\gamma)\right] = \frac{v_o^2\omega\eta'}{2[(\eta')^2 + (\eta'')^2]^{1/2}} \tag{5}$$

The simplest possible linear viscoelastic fluid is the Maxwell model:

$$\tau + \lambda\dot\tau = \eta_0\dot\gamma \tag{6}$$

for which

$$\eta' = \frac{\eta_0}{1+\omega^2\lambda^2}, \quad \eta'' = \frac{\omega\lambda\eta_0}{1+\omega^2\lambda^2} \tag{7}$$

For this case, Eq. 5 becomes

$$\dot{e} = \frac{v_o^2\omega}{2\sqrt{1+\omega^2\lambda^2}} \tag{8}$$

Since the Maxwell fluid reduces to a purely viscous Newtonian fluid for a relaxation time (λ) of zero, it is evident from Eq. 8 that elastic properties (represented by λ) serve to reduce the rate of energy dissipation relative to that of an inelastic fluid by a factor equal to the radical in the denominator.

In order to apply these results to turbulent flow, we consider turbulence to consist of a spectrum of oscillating eddies with amplitude v_o' and frequency ω, following Davies (1972). The local mean rate of energy dissipation by the eddies per unit mass in a viscous fluid (e.g., the solvent) is given by

$$\dot{e}_s = \frac{\overline{\tau_e^\dagger \dot{\gamma}_e^\dagger}}{\rho} \tag{9}$$

where the turbulent eddy (Reynolds) stress and equivalent turbulent shear rate are given by:

$$\tau_e^\dagger = \rho(\bar{v}')^2, \quad \dot{\gamma}_e^\dagger = \bar{v}'/l_e = \omega \tag{10}$$

Equation 9 thus becomes

$$\dot{e}_s = \bar{v}'^2\omega = \frac{1}{2}(v_o')^2\omega \tag{11}$$

where $\bar{v}'$ is the rms eddy velocity, i.e., $\bar{v}' = v_o'/\sqrt{2}$. Now according to Davies (1972), the maximum rate of energy dissipation by turbulent eddies in a viscous fluid occurs when the eddy Reynolds number is about 9, i.e.

$$N'_{Re} = \frac{l_e\bar{v}'}{\nu} = \frac{(\bar{v}')^2}{\omega\nu} \simeq 9 \tag{12}$$

or

$$(\bar{v}')^2 \simeq 9\omega\nu \tag{13}$$

Equating the total rate of energy dissipation from Eq. 4 to the eddy dissipation from Eq. 11, and using Eq. 13 to eliminate $\bar{v}'$, provides a relation between the friction factor and the dissipative eddy frequency for a purely viscous fluid (solvent):

$$f_s = \frac{\dot{e}D}{2v^3} = \frac{9}{2}\left(\frac{D\omega}{v}\right)^2\left(\frac{1}{N_{\rm Res}}\right) \tag{14}$$

Rearranging Eq. 14 gives the following expression for the most dissipative eddy frequency:

$$\omega = (2f_sN_{\rm Res}/9)^{1/2}v/D \tag{15}$$

For a Newtonian fluid in a smooth tube, the friction factor is given by the Blasius equation:

$$f_s = \frac{0.0791}{N_{\rm Res}^{1/4}} \tag{16}$$

Substituting Eq. 16 into Eq. 14, we get

$$\omega = 0.0166\left(\frac{8v}{D}\right)N_{\rm Res}^{3/8} \tag{17}$$

for the frequency of the dissipative eddies. For a Reynolds number of about 5×10^4, Eq. 17 reduces to $\omega \simeq 8v/D$.

For the viscoelastic polymer solution, the rate of energy dissipation by the eddies is modified by the elastic properties according to Eq. 8. Using this expression instead of Eq. 11 in Eq. 14 results in the following expression for the polymer solution friction factor

$$f_p = \frac{f_s}{\sqrt{1+\lambda^2\omega_p^2}} = \frac{9}{2}\left(\frac{D\omega_p}{v}\right)^2\frac{1}{N_{\rm Res}\sqrt{1+\lambda^2\omega_p^2}} \tag{18}$$

If it is assumed that the dissipative eddy frequency for the polymer solution is related to the solution friction factor and Reynolds number by an expression analogous to Eq. 14, i.e.

$$\omega_p = (2f_pN_{\rm Rep}/9)^{1/2}v/D \tag{19}$$

substitution of f_p from Eq. 18 into Eq. 19 gives the following relation between ω and ω_p:

$$\omega_p = \frac{\omega}{(1+\lambda^2\omega_p^2)^{1/4}}\left(\frac{N_{\rm Rep}}{N_{\rm Res}}\right)^{1/2} \tag{20}$$

where

$$N_{\rm Rep} = \frac{Dv\rho}{\eta_p}$$

This indicates that the spectrum of the dissipative eddies in the polymer solution should be shifted to slightly lower frequencies as a consequence of a finite relaxation time, as well as a consequence of a higher viscosity, since $N_{\rm Rep} \leq N_{\rm Res}$. This is consistent with previous experimental observations (Baker, 1973; Hanratty and Chorn, 1977; Fortuna and Hanratty, 1971). In general, the values of ω and ω_p are not greatly different.

Equation 18 relates the Newtonian (solvent) friction factor, f_s, to the corresponding friction factor for the polymer solution, f_p. This can be written

$$f_p = \frac{f_s}{\sqrt{1+N_{De}^2}} \tag{21}$$

where $N_{De} = \omega_p\lambda$ is the characteristic dimensionless eddy frequency (Deborah number) for the system. Both ω_p and ω are dependent upon the rheological properties of a given polymer solution and can be evaluated as follows.

The appropriate dissipative eddy frequency is assumed to be given by Eq. 20, in which the solvent frequency, ω, is given by Eq. 17. However, for simplicity, the ω_p in the denominator of Eq. 20 may be replaced by $(8v/D)$, since these are of the same magnitude. From Eq. 17, it is seen that ω is equal to $8v/D$ at a typical value of the Reynolds number of about 5×10^4 and ω_p does not differ greatly from ω. It was found that this approximation had a negligible effect on the final results. Also, the ratio $(N_{\rm Rep}/N_{\rm Res}) = \mu_s/\eta_p$ is the ratio of the solvent to polymer solution viscosity. However, since the solution is non-Newtonian, η_p is not a parameter, but a function whose maximum value is η_0. If it is assumed that the appropriate characteristic viscosity should be that associated with the force required to deform the polymer molecule from their equilibrium configuration, or conversely, the maximum force resisting the rate at which equilibrium configuration could be achieved, then the zero shear viscosity should be the appropriate parameter. With these approximations, then, the assumed form for N_{De} becomes:

$$N_{De} = \frac{0.0166(8v\lambda/D)N_{\rm Res}^{3/8}(\mu_s/\eta_0)^{1/2}}{[1+(8v\lambda/D)^2]^{1/4}} \tag{22}$$

The other question which must be resolved is that of the appropriate evaluation of λ. Since our simple model for energy dissipation is based on the linear Maxwell model (Eq. 6), a constant value for λ is implied. However, the polymer solutions are not linear viscoelastic fluids, as illustrated by Figures 5 and 6. Indeed, linear models such as Eq. 6, are valid, at most, for infinitessimal deformations, although they may be generalized to large deformations or continuous flow by replacing the time derivative by a suitable corotation convected derivative operator (the Jaumann derivative), as follows:

$$\tau_{ij} + \lambda\frac{\mathscr{D}\tau_{ij}}{\mathscr{D}t} = \eta_0\Delta_{ij} \tag{23}$$

This model is not only more general, but predicts nonlinear properties as well. For example, the corresponding viscosity function from Eq. 23 is:

$$\eta = \frac{\eta_0}{1+\lambda^2\dot{\gamma}^2} \tag{24}$$

which exhibits qualitatively correct shear-thinning behavior. A slight modification to Eq. 23 can be made, to account for the observed high shear limiting viscosity, η_∞, as follows:

$$\tau_{ij} + \lambda\frac{\mathscr{D}\tau_{ij}}{\mathscr{D}t} = \eta_0\Delta_{ij} + \lambda\eta_\infty\frac{\mathscr{D}\Delta_{ij}}{\mathscr{D}t} \tag{25}$$

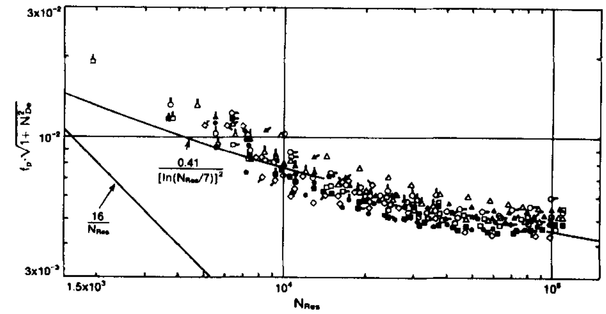

Figure 7. Generalized friction factor vs. solvent Reynolds number. Points shown include entire range of data. For clarity, not all data points are shown.

This model is also sometimes known as the Jeffrey's model, and predicts the following viscosity function:

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1 + \lambda^2\dot{\gamma}^2)} \tag{26}$$

Comparison of Eqs. 1 and 26 shows that the Jeffrey's model viscosity function is qualitatively similar to that for our polymer solutions. In these equations, the significance of λ is still the same, i.e., the characteristic relaxation time, which we have related through our simplified model to energy dissipation in turbulent eddies. In fact, by equating the viscosity functions in Eqs. 1 and 6, we can solve for λ, thus relating the characteristic time constant to the measured properties of our solutions:

$$\lambda = [(1 + \zeta^2\dot{\gamma}^2)^\Omega - 1]^{1/2}/\dot{\gamma} \tag{27}$$

For application to turbulence, we have seen that the equivalent eddy shear rate is ω, which is of the order of $(8v/D)$. Making this substitution in Eq. 27 results in the following expression:

$$\frac{8v\lambda}{D} = \left\{\left[1 + \left(\frac{8v\zeta}{D}\right)^2\right]^\Omega - 1\right\}^{1/2} \tag{28}$$

If $(8v\zeta/D)^2 \gg 1$, which is often the case, this reduces to:

$$\frac{8v\lambda}{D} \simeq \left(\frac{8v\zeta}{D}\right)^\Omega = \left(\frac{8v\zeta}{D}\right)^{(1-n)/2} \tag{29}$$

It is seen (from Eqs. 1 and 26) that the nonlinearity in the viscosity function arises from the same fluid characteristics that describe their elastic behavior, i.e., λ or ζ. This is also illustrated by Eq. 2 for the normal stresses, and is the principle behind the development of Eq. 3 relating the normal stress and viscosity functions. This result is really quite fortuitous, since it enables the determination of elastic properties (e.g., ζ) from appropriate characteristics of the viscosity function. In this case, ζ is the reciprocal of the value of $\dot{\gamma}$ where the power law region of the viscosity function intersects with η_0.

DATA CORRELATION

Because of the approximations and assumptions involved in the model leading to Eq. 22 for N_{De}, it would not be expected to provide an accurate quantitative representation of the data. However, it can be used as a guide for correlation of data in the form:

$$N_{De} = \frac{A(8v\lambda/D)N_{\text{Res}}^b}{[1 + (8v\lambda/D)^2]^c}\left(\frac{\mu_s}{\eta_0}\right)^d \tag{30}$$

in which the parameters A, b, c, and d can be determined from friction loss data by regression analysis. This was done using Eq. 28 for $(8v\lambda/D)$, with the following result:

$$N_{De} = \frac{0.0867(8v\lambda/D)N_{\text{Res}}^{0.34}}{[1 + (8v\lambda/D)^2]^{0.33}}\left(\frac{\mu_s}{\eta_0}\right)^{0.237} \tag{31}$$

This expression enabled all of the data for all concentrations of both fresh and sheared solutions in all tube sizes tested to be correlated on the same curve which represents the classical Fanning friction factor vs. solvent Reynolds number, with a correlation coefficient (R^2) of 0.956. This correlation is shown in Figure 7. The solid line on the plot represents the Colebrook Eq. (1939) for smooth tubes, which is quite close to, but slightly more accurate than the Blasius equation. The fact that the values of the empirical exponential parameters in Eq. 31, which were determined by regression analysis, are comparable to those in Eq. 22 resulting from the simplified model approach, is very encouraging and lends credance to the result.

Because the values of the exponential parameters in Eq. 31 are nearly the same, an alternative form which includes only two empirical parameters was also evaluated by regression analysis with the following result:

$$N_{De} = 0.145\left(\frac{8v\lambda}{D}\right)\left[\frac{N_{\text{Res}}}{1 + \left(\frac{8v\lambda}{D}\right)^2}\frac{\mu_s}{\eta_0}\right]^{0.285} \tag{32}$$

The R^2 correlation coefficient for this expression is 0.952, and it contains only two empirical parameters.

DISCUSSION

We have shown how a simplified concept of reduced energy dissipation in the turbulent flow of a viscoelastic fluid, coupled with the proper definition of the fluid relaxation time consistent with the observed nonlinear rheological properties of these solutions, enables the definition of the friction factor to be extended to include a dependence upon a dimensionless relaxation time parameter (Deborah number) which properly reflects the influence of fluid elasticity and viscosity upon turbulent drag reduction. The resulting correlation based upon these concepts requires only a knowledge of the pertinent elastic and viscous rheological parameters for the solutions, all of which can be obtained directly from a complete characterization of the apparent viscosity function. The solutions used in this study were "concentrated," i.e., more concentrated than necessary for maximum drag reduction, for two reasons: (a) these solutions exhibited very pronounced drag reduction, which varied with tube size, polymer concentration, and degradation; and (b) the nonlinear viscous and elastic properties of these solutions were sufficiently pronounced to enable a quantitative characterization of these properties by conventional methods. However, significant drag reduction can be achieved at considerably lower concentrations of these same polymers, and many studies have been concerned with such dilute solutions, for which characterization of the true rheological properties is much more difficult.

In fact, sufficiently dilute solutions often appear to be essentially Newtonian in their viscous behavior, and furthermore tend to exhibit a dependence of drag reduction on concentration and degradation which is just the opposite to that exhibited by these more concentrated solutions. Hence a discussion of the effects of concentration and degradation in light of the proposed model for drag reduction is appropriate.

The degree of drag reduction has been shown to increase with N_{De}, which in turn depends upon the viscosity (η_0) and the elasticity (λ or ζ) of the solutions. From Eq. 27, it is seen that $\lambda \sim \zeta^\Omega$, where $\Omega \sim 0.25$ (from Table 1). From either the model formulation (Eq. 22) or the final correlation (Eq. 31), N_{De} is seen to be proportional to $(\zeta/\eta_0^4)^k$, where $k = 1/8$ from the model and $1/12$ from the correlation. Thus the degree of drag reduction increases with an increase in ζ and decreases with an increase in η_0, but there is a fourth order greater sensitivity to changes in η_0 than to changes in ζ. Thus the relative changes in ζ and η_0 resulting from changes in concentration and from degradation may explain the corresponding changes in drag reductions as follows.

From Table 1, it is seen that as concentration decreases for the concentrated solutions, η_0 decreases and ζ increases. Both of these effects should increase the degree of drag reduction, as observed. However, for dilute solutions the result must be quite different, since η_0 must approach μ_s (the solvent viscosity) and ζ must decrease to zero as the concentration approaches zero. Therefore, there must be a concentration for which ζ is maximum (presumeably that for which drag reduction is maximum), although η_0 decreases monotonically as concentration decreases. Although decreasing ζ will decrease the degree of drag reduction and decreasing η_0 will increase the degree of drag reduction, the former is more significant for dilute solutions since relative changes in η_0 are limited by the approach to a finite limit (μ_s), whereas relative changes in ζ are not bounded since the limiting value is zero. Thus it would be expected that the drag reducing effectiveness of dilute solutions would decrease with decreasing concentration, as is normally observed. Indeed, it is postulated that for polymer solutions of all concentrations, $\eta_0 > \mu_s$ and $\zeta > 0$. However, for very dilute solutions, the difference between η_0 and μ_s is very small and hence difficult to measure. Likewise there must still be a non-Newtonian region to the viscosity curve, although the magnitude of viscosity difference is small and the transition (the shear rate corresponding to $1/\zeta$) occurs at ever higher shear rates as the concentration decreases. Thus measurements required to determine these parameters for dilute solutions have to be quite sensitive and would indeed be much more difficult.

The effect of degradation on drag reduction for dilute and concentrated solutions can also be explained in terms of the relative changes in the parameters η_0 and ζ. For the concentrated solutions, it is seen from Table 1 that both η_0 and ζ are lower for the degraded solutions. In fact, the decrease in ζ is even greater than the decrease in η_0. However, this greater decrease in ζ is still not sufficient to offset the much greater (fourth order) sensitivity of N_{De} to η_0, so the net effect is an increase in drag reduction with degradation for these concentrated solutions. For the more dilute solutions, however, the possible relative change in η_0 due to degradation is limited by the finite minimum value of μ_s, whereas the relative change in ζ is not bounded by a finite limit. Therefore, for dilute solutions, it is quite possible that reductions in ζ may more than offset the greater sensitivity to corresponding changes in η_0, thus reducing the degree of drag reduction with degradation as has been observed for such solutions.

CONCLUSIONS

A correlation has been developed based upon the concept of energy dissipation in viscoelastic fluids, which enables the prediction of friction loss of "concentrated" fresh and shear degraded polymer solutions of several concentrations in a wide range of tube sizes. The correlation involves a generalized definition of the Fanning friction factor (Eq. 21) which accounts for the effect of viscoelastic properties on friction loss, and enables friction factor data for drag reducing fluids to be represented by the same classical correlations that represent ordinary Newtonian fluids. Only three parameters characteristic of the solution properties are required: the zero shear viscosity, a characteristic time constant, and the power law flow index, each of which can be obtained from the apparent viscosity curve. The correlation is applied as follows: The time constant, ζ, and the flow index, n, (or $\Omega = (1 - n)/2$) are used in Eq. 28 to calculate $(8v\lambda/D)$. This, along with η_0, is used in Eq. 31 to determine N_{De}, which is then used with Eq. 21 to relate the Fanning friction factor of the polymer solution to that of the Newtonian solvent at the solvent Reynolds number. The effect of degradation and changes in concentration on drag reduction can also be explained in terms of corresponding changes in the parameters η_0 and ζ.

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NOTATION

D = tube diameter, m
$\mathcal{D}/\mathcal{D}t$ = Jaumann or corotational time derivative, s^{-1}
$\dot{e}$ = rate of energy dissipation per unit mass, W/kg
f = Fanning friction factor
l = characteristic length, m
L = tube length, m
n = power law flow index
N_{De} = Deborah number = $\omega\lambda$
N_{Re} = Reynolds number
ΔP = pressure change, Pa
Q = volumetric flow rate, m^3/s
$\bar{v}'$ = root mean squared velocity fluctuation, m/s
v_o' = amplitude of velocity fluctuation, m/s
v = average tube velocity, m/s
v_o = amplitude of oscillating velocity, m/s
V = volume, m^3
β = parameter in Eq. 2
$\dot{\gamma}$ = shear rate, s^{-1}
Δ_{ij} = rate of strain tensor, s^{-1}
ζ = time constant of polymer solution, s
ν = kinematic viscosity, m^2/s
ρ = density, kg/m^3
η = apparent viscosity, Pa·s
η' = real part of complex viscosity, Pa·s
η'' = imaginary part of complex viscosity, Pa·s
η_0 = zero shear rate limiting viscosity, Pa·s
η_∞ = high shear rate limiting viscosity, Pa·s
θ = parameter in Eq. 2, s
λ = relaxation time, s
μ = Newtonian viscosity, Pa·s
τ = shear stress, Pa
τ_{11} = normal stress in direction of motion, Pa
τ_{22} = normal stress in direction of gradient, Pa
τ_{ij} = shear stress tensor, Pa
ψ_1 = first normal stress difference function, $Pa \cdot s^2$
ω = frequency of oscillation, s^{-1}
Ω = parameter in Eq. 1

Subscripts

e = eddy
p = polymer solution
s = solvent
w = wall value

Superscript

= eddy property (except in Eq. 3)

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Modeling of Growth Rate Dispersion of Citric Acid Monohydrate in Continuous Crystallizers

A mathematical model for prediction of the crystal size distribution from a continuous crystallizer is presented. The kinetic data used for the model were obtained from batch contact nucleation experiments with citric acid monohydrate. In these experiments, the distribution of growth rates as well as the initial size distribution were estimated. Results from the model indicate that the excess number of crystals usually present at small sizes in continuous crystallizers is due to growth rate dispersion (where crystals of the same size may have different growth rates) and not size dependent growth.

K. A. BERGLUND and
M. A. LARSON
Department of Chemical Engineering
Iowa State University
Ames, IA 50011

SCOPE

The population balance technique developed by Randolph and Larson (1971) has been used extensively for both kinetic measurement and modeling of continuous mixed suspension, mixed product removal (MSMPR) crystallizers. When the assumptions of size independent crystal growth, all crystals with equal growth (i.e., no growth rate dispersion) and all nuclei formed at a near zero size are invoked, a semilogarithmic relation is predicted between crystal population density and size. Experimental evidence from continuous crystallizers, however, has shown that at lower crystal sizes (<20 μm) orders of magnitude more crystals are present than are predicted by this relation. Clearly any or all of the assumptions may be in error.

The importance of the deviation from the model is that the semilogarithmic population density-crystal size plot is used to determine kinetic data. When the model holds and a straight line is produced, the growth rate is determined from the slope and the nucleation rate is determined from both the slope and intercept. When curvature occurs, the slope no longer has a single value and the intercept must be determined by some means of nonlinear extrapolation. In order to develop unambiguous kinetic models, it is necessary to understand the causes for the curvature.

The present study made use of data taken previously (Berglund, and Larson, 1982) in contact nucleation experiments with the citric acid monohydrate-water system. These data suggest that size independent growth rate, growth rate dispersion, and initial size distribution are present. Use was then made of probability transform techniques to develop a model for a continuous MSMPR crystallizer that accounts for these phenomena. Studies with the model concentrated on the effects of growth rate distribution, initial size distribution, and the interaction between the two distributions on the product population density.

Correspondence concerning this paper should be directed to K. A. Berglund, Department of Chemical Engineering, Michigan State University, East Lansing, MI 48824